Numerical Integration (Gauss-Legendre Two Point Formula)

Compiled by

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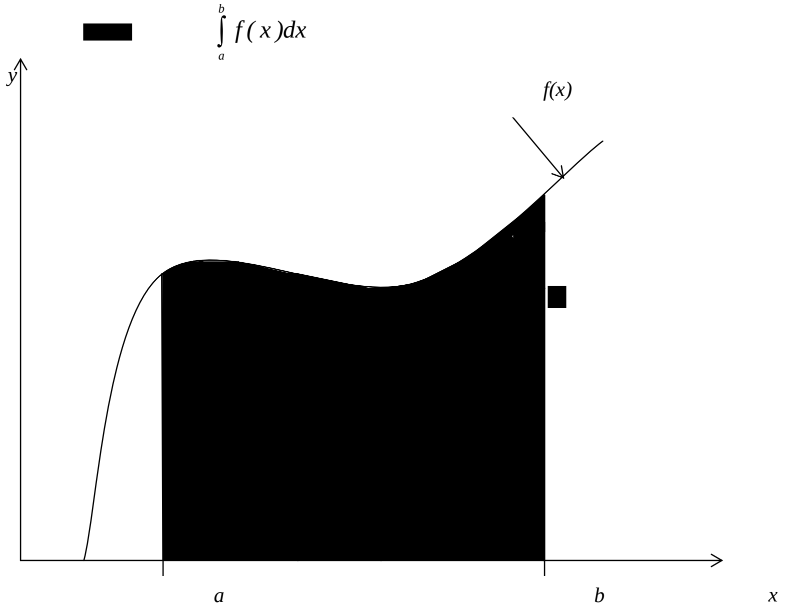
What is Integration ?

- The process of measuring the area under a function plotted on a graph.

$$I = \int_a^b f(x)dx$$

- Where:
- *f(x)* is the integrand
- a= lower limit of integration
- b= upper limit of integration

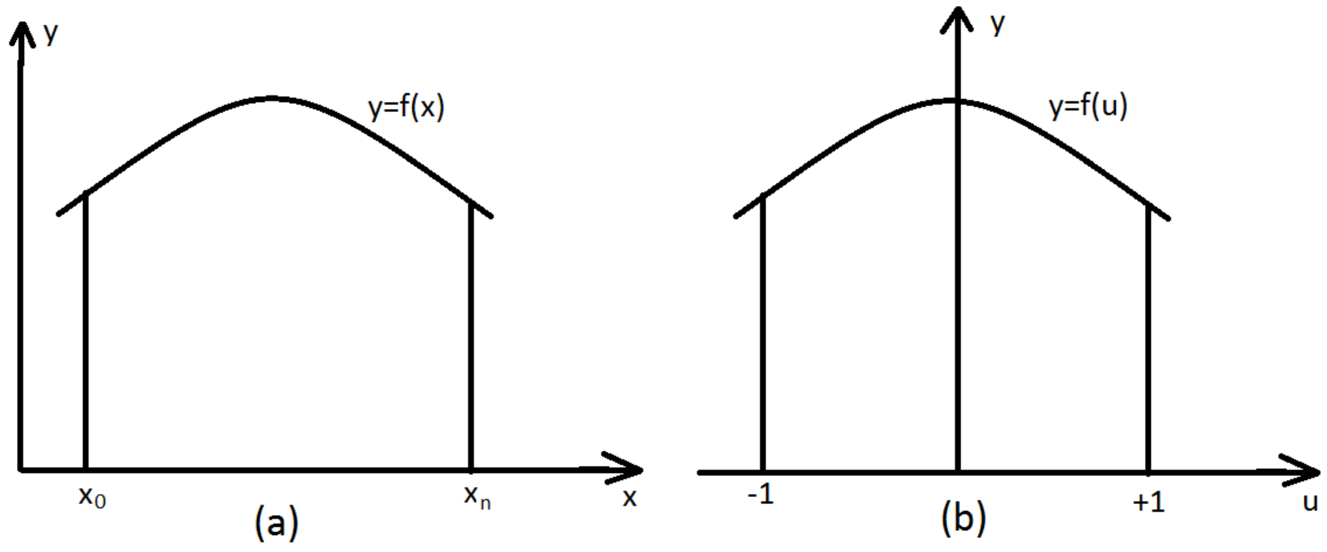


Limitations of Newton-Cotes Formula

- When we calculate integration by Newton-Cotes method, the area is divided in number of strips with step size h.
- The formula includes y values (function values) at the corresponding x values.
- But for complicated equations it becomes difficult to find function values.
- As more processing time is required, computer program also becomes inefficient.
- If we reduce the number of strips, number of function values will be less. But, this affects the accuracy of the solution.
- This limitation of Newton-Cotes equation is overcome by Gauss-Legendre quadrature method.

Gauss-Legendre Quadrature Method

This method converts the function variable f(x) to a function variable f(u) such that the original limits x_0 and x_n gets changed to (-1) and (+1) respectively whereas the area under the curve f(x) and f(u) remains same. Refer figure (a) and (b)



Gauss-Legendre Quadrature Method …..Contd

For figure (a) the function is written as, $I = \int_{x_0}^{x_n} f(x)dx$

Similarly for figure (b) the function is written as, $I = \int_{-1}^{+1} f(u)du$

Assuming linear relationship between x and u, let the relation be,

$$x = Cu + d \text{…………………. (1)}$$

From figures (a) & (b), at $x = x_0$, u=-1

$$x_0 = -C + d \text{ …………………. (2)}$$

at $x = x_n$, u=1

$$x_n = C + d \text{ …………………. (3)}$$

Solving eq. (2) & (3), we get

$$C = \frac{x_n - x_0}{2} \;\&\; d = \frac{x_n + x_0}{2}$$

Gauss-Legendre Quadrature Method .....Contd

Now differentiating eq. (1) $dx = C.du$

The transformation from f(x) to f(u) can be obtained by substituting $x = Cu + d$ and $dx = C.du$ in the given function

$$\int_{x_0}^{x_n} f(x)dx = \int_{-1}^{+1} f(Cu + d).C.du$$

$$C\int_{-1}^{+1} f(Cu + d)du$$

$$C = \frac{x_n - x_0}{2} \text{ \& } d = \frac{x_n + x_0}{2}$$

As we have transferred f(x) equivalent to f(u), the problem has reduced to find area bounded by y=f(u), u=-1, u=+1 and y=0. The solution can be given in series form as,

$$I = \int_{-1}^{+1} f(u)du = \sum_{i=0}^{n} \lambda_i.f(u_i) \ldots\ldots\ldots..(4)$$

where λ_i=weights of integration.

As per the accuracy level, we can consider few terms of the series solution. The obtained equation are called as two point formula and three point formula.

Gauss-Legendre Two Point Formula

For two point formula first two terms of series solution are considered. Hence eq. (4) i.e. $\int_{-1}^{+1} f(u)du = \sum_{i=0}^{n} \lambda_i . f(u_i)$ becomes,

$$\int_{-1}^{+1} f(u)du = \lambda_0 . f(u_0) + \lambda_1 . f(u_1) \ldots \ldots \ldots .. (5)$$

To find the value of four unknown λ_0, λ_1, u_0 and u_1 four equations are required. To find four unknown we substitute f(u) by $1, u, u^2$ and u^3 respectively

(a) for $f(u) = 1,\ f(u_0) = 1\ and f(u_1) = 1, substitute\ in\ eq. (5)$

$$\int_{-1}^{+1} 1. du = \lambda_0 . 1 + \lambda_1 . 1$$

$$2 = \lambda_0 + \lambda_1 \ldots \ldots \ldots .. (6)$$

(b) for $f(u) = u,\ f(u_0) = u_0 and f(u_1) = u_1, substitute\ in\ eq. (5)$

$$\int_{-1}^{+1} u. du = \lambda_0 . u_0 + \lambda_1 . u_1$$

$$0 = \lambda_0 . u_0 + \lambda_1 . u_1 \ldots \ldots \ldots .. (7)$$

Gauss-Legendre Two Point Formula .....Contd

(c) for $f(u) = u^2, f(u_0) = u_0^2\ and f(u_1) = u_1^2$,*substitute in eq. (5)*

$$\int_{-1}^{+1} u^2.du = \lambda_0.u_0^2 + \lambda_1.u_1^2$$

$$\frac{2}{3} = \lambda_0.u_0^2 + \lambda_1.u_1^2 \ldots\ldots\ldots..(8)$$

(d) for $f(u) = u^3, f(u_0) = u_0^3\ and f(u_1) = u_1^3$,*substitute in eq. (5)*

$$\int_{-1}^{+1} u^3.du = \lambda_0.u_0^3 + \lambda_1.u_1^3$$

$$0 = \lambda_0.u_0^3 + \lambda_1.u_1^3 \ldots\ldots\ldots..(9)$$

Solving eq. (6), (7), (8) and (9) we get

$$\lambda_0 = 1, \lambda_1 = 1, u_0 = \left(\frac{-1}{\sqrt{3}}\right) \text{and } u_1 = \left(\frac{1}{\sqrt{3}}\right)$$

Substituting these values in eq. (5)

$$\int_{-1}^{+1} f(u)du = 1.f\left(\frac{-1}{\sqrt{3}}\right) + 1.f\left(\frac{1}{\sqrt{3}}\right)$$

Gauss-Legendre Two Point Formula .....Contd

Which can be written as,

$$\int_{x_0}^{x_n} f(x)dx = \int_{-1}^{+1} f(u)du = f(\frac{-1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})$$

The above equation is called as Gauss-Legendre Two Point Formula.

Gauss-Legendre Two Point Formula .....Contd

To find the integration by Gauss-Legendre Two Point Formula use following steps

1) Write the given function in the form of $I = \int_{x_0}^{x_n} f(x)dx$
2) Calculate the values of 'C' and 'd' by using relation $C = \frac{x_n - x_0}{2}$ & $d = \frac{x_n + x_0}{2}$
3) Calculate the two values of x i.e. x_1 and x_2 and corresponding values of y by using relation,

$$x_1 = C\left(\frac{1}{\sqrt{3}}\right) + d \quad \text{and } x_2 = -C\left(\frac{1}{\sqrt{3}}\right) + d$$

$$y_1 = f(x_1) \quad \text{and } \; y_2 = f(x_2)$$

4) Calculate the area under the curve by using

$$A = (y_1 + y_2).C$$

Gauss-Legendre Two Point Formula .....Contd

Example: Compute the integral $I = \int_{-2}^{2}(e^{-x/2})dx$ using Gauss two point formula.

Solution: $f(x) = (e^{-x/2})$, x_0=-2, x_n=2

Step 1: Calculate the value of C and d

$$C = \frac{x_n - x_0}{2} = \frac{2-(-2)}{2} = 2$$

$$d = \frac{x_n + x_0}{2} = \frac{2+(-2)}{2} = 0$$

Step 2: Calculate the value of x_1 *and* x_2

$$x_1 = C\left(\frac{1}{\sqrt{3}}\right) + d = \left(\frac{2}{\sqrt{3}}\right) + 0 = 1.1547$$

$$x_2 = -C\left(\frac{1}{\sqrt{3}}\right) + d = \left(\frac{-2}{\sqrt{3}}\right) + 0 = -1.1547$$

Gauss-Legendre Two Point Formula .....Contd

Step 3: Calculate the value of $\boldsymbol{y}$, $\boldsymbol{y = \mathrm{f}(x) = (e^{-x/2})}$

$$y_1 = \left(e^{-x_1/2}\right) = e^{-1.1547/2} = 0.56138$$

$$y_2 = \left(e^{-x_2/2}\right) = e^{-(-1.1547)/2} = 1.7813$$

Step 4: Calculate the area under the curve (A)

$$A = (y_1 + y_2).C = (0.56138 + 1.7813)\mathrm{x}2 = 4.6853$$

$$\mathrm{I} = \int_{-2}^{2} (e^{-x/2})dx = 4.6853$$

Problems on Gauss-Legendre two point formula

1. Compute the integral $\int_0^1 (4 + 2cosx)dx$ using Gauss-Legendre two point formula.
2. Find the integration $\int_0^2 (x^2 - 3x + 2)dx$ using Gauss-Legendre two point formula.
3. Evaluate $\int_0^1 (xe^x)dx$ using Gauss-Legendre two point formula.
4. Find the integration $x^3 + x - 1$ with limits 1 to 4 using Gauss-Legendre two point formula.

Problems on Gauss-Legendre two point formula .....Contd

5. Compute the integral $\int_0^1 (\frac{cosx-x}{1+x})dx$ using Gauss-Legendre two point formula.
6. Compute the integral $\int_2^3 (\frac{x^3-4x+2}{lnx})dx$ using Gauss-Legendre two point formula.
7. Compute the integral $\int_{-1}^{1} (x^3 - 6x + 13)dx$ using Gauss-Legendre two point formula.

Reference Books

1. Steven C. Chapra, Raymond P. Canale, Numerical Methods for Engineers, 4/e, Tata McGraw Hill Editions

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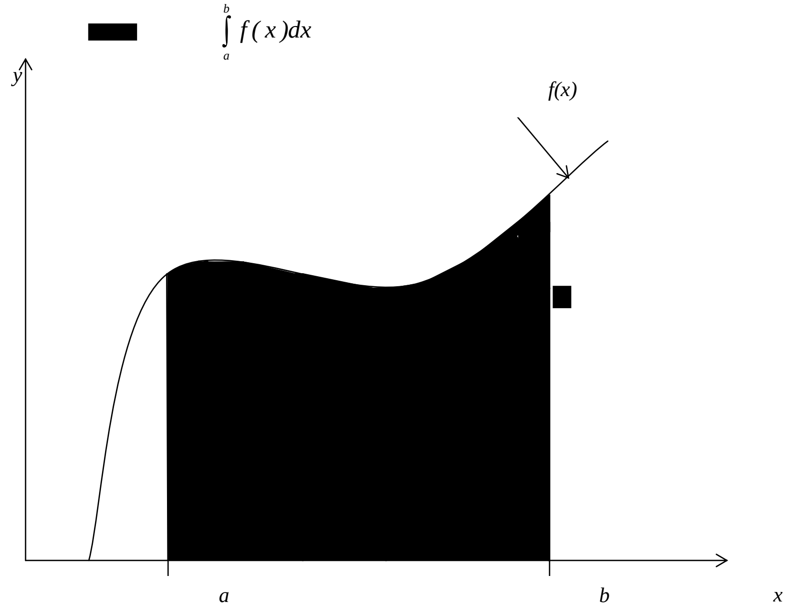

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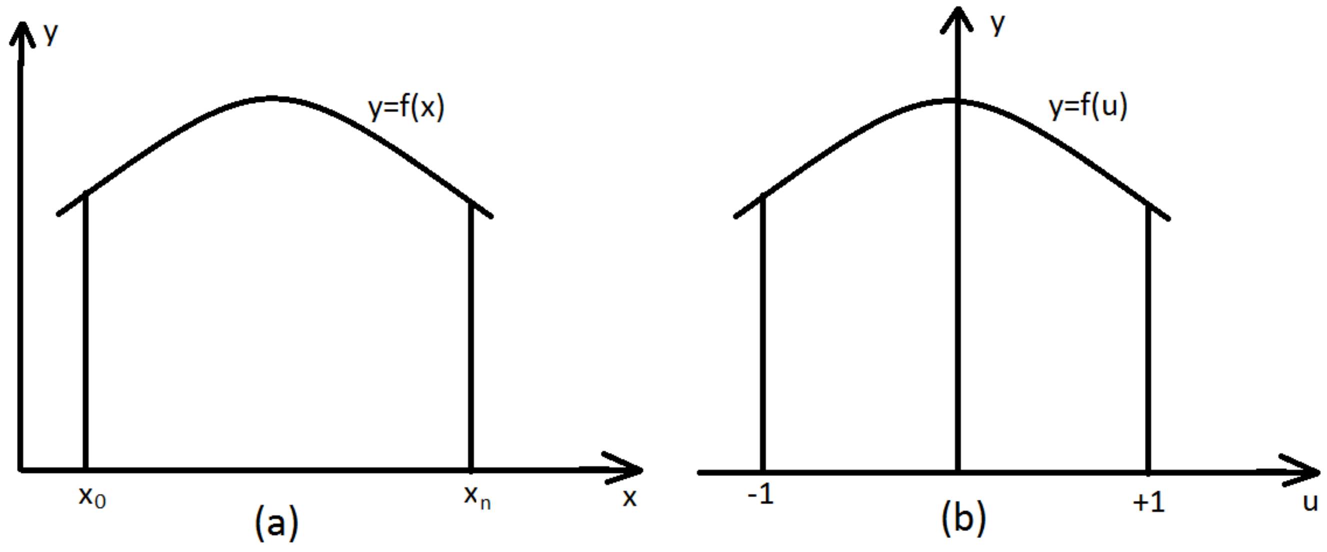

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Solving eq. (2) & (3), we get

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Gauss-Legendre Quadrature Method …..Contd

Now differentiating eq. (1) $dx = C.du$

The transformation from f(x) to f(u) can be obtained by substituting $x = Cu + d$ and $dx = C.du$ in the given function

$$\int_{x_0}^{x_n} f(x)dx = \int_{-1}^{+1} f(Cu + d).C.du$$

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$$C = \frac{x_n - x_0}{2} \;\&\; d = \frac{x_n + x_0}{2}$$

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$$I = \int_{-1}^{+1} f(u)du = \sum_{i=0}^{n} \lambda_i . f(u_i) \ldots\ldots\ldots..(4)$$

where λ_i=weights of integration.

As per the accuracy level, we can consider few terms of the series solution. The obtained equation are called as Three Point formula and three point formula.

Gauss-Legendre Three Point Formula

For three point formula first three terms of series solution are considered. Hence eq. (4) i.e. $\int_{-1}^{+1} f(u)du = \sum_{i=0}^{n} \lambda_i . f(u_i)$ becomes,

$$\int_{-1}^{+1} f(u)du = \lambda_0 . f(u_0) + \lambda_1 . f(u_1) + \lambda_2 . f(u_2) \ldots \ldots \ldots . . (5)$$

To find the value of six unknown $\lambda_0, \lambda_1, \lambda_2$ u_0, u_1 and u_2 six equations are required. To find six unknown we substitute f(u) by $1, u, u^2, u^3, u^4 and\ u^5$ respectively

(a) for $f(u) = 1, f(u_0) = 1, f(u_1) = 1 and f(u_2) = 1, substitute\ in\ eq. (5)$

$$\int_{-1}^{+1} 1. du = \lambda_0 . 1 + \lambda_1 . 1 + \lambda_2 . 1$$

$$2 = \lambda_0 + \lambda_1 + \lambda_2 \ldots \ldots \ldots . . (6)$$

(b) for $f(u) = u, f(u_0) = u_0, f(u_1) = u_1 and f(u_2) = u_2, substitute\ in\ eq. (5)$

$$\int_{-1}^{+1} u. du = \lambda_0 . u_0 + \lambda_1 . u_1 + \lambda_2 . u_2$$

$$0 = \lambda_0 . u_0 + \lambda_1 . u_1 + \lambda_2 . u_2 \ldots \ldots \ldots . . (7)$$

Gauss-Legendre Three Point Formula ..Contd

(c) for $f(u) = u^2, f(u_0) = u_0^2, f(u_1) = u_1^2 and f(u_2) = u_2^2$*, substitute in eq. (5)*

$$\int_{-1}^{+1} u^2.du = \lambda_0.u_0^2 + \lambda_1.u_1^2 + \lambda_2.u_2^2$$

$$\frac{2}{3} = \lambda_0.u_0^2 + \lambda_1.u_1^2 + \lambda_2.u_2^2 \ldots\ldots\ldots..(8)$$

(d) for $f(u) = u^3, f(u_0) = u_0^3, and f(u_1) = u_1^3 and f(u_2) = u_2^3$*, substitute in eq. (5)*

$$\int_{-1}^{+1} u^3.du = \lambda_0.u_0^3 + \lambda_1.u_1^3 + \lambda_2.u_2^3$$

$$0 = \lambda_0.u_0^3 + \lambda_1.u_1^3 + \lambda_2.u_2^3 \ldots\ldots\ldots..(9)$$

(e) for $f(u) = u^4, f(u_0) = u_0^4, and f(u_1) = u_1^4 and f(u_2) = u_2^4$*, substitute in eq. (5)*

$$\int_{-1}^{+1} u^4.du = \lambda_0.u_0^4 + \lambda_1.u_1^4 + \lambda_2.u_2^4$$

$$\frac{2}{5} = \lambda_0.u_0^4 + \lambda_1.u_1^4 + \lambda_2.u_2^4 \ldots\ldots\ldots..(10)$$

Gauss-Legendre Three Point Formula ..Contd

(f) for $f(u) = u^5, f(u_0) = u_0^5, and f(u_1) = u_1^5 and f(u_2) = u_2^5, substitute$ $in\ eq.\ (5)$

$$\int_{-1}^{+1} u^5.du = \lambda_0.u_0^5 + \lambda_1.u_1^5 + \lambda_2.u_2^5$$

$$0 = \lambda_0.u_0^5 + \lambda_1.u_1^5 + \lambda_2.u_2^5 \ldots\ldots\ldots..(11)$$

Solving eq. (6), (7), (8), (9), (10) and (11) we get

$$\lambda_0 = \left(\frac{5}{9}\right), \lambda_1 = \left(\frac{8}{9}\right), \lambda_1 = \left(\frac{5}{9}\right), u_0 = \left(-\sqrt{\frac{3}{5}}\right), u_1 = 0 \text{ and} u_2 = \left(\sqrt{\frac{3}{5}}\right)$$

Substituting these values in eq. (5)

$$\int_{-1}^{+1} f(u)du = \frac{5}{9}.f(-\sqrt{\frac{3}{5}}) + \frac{8}{9}.f(0) + \frac{5}{9}.f(\sqrt{\frac{3}{5}})$$

Gauss-Legendre Three Point Formula ..Contd

Which can be written as,

$$\int_{x_0}^{x_n} f(x)dx = \int_{-1}^{1} f(u)du = \frac{5}{9}.f(-\sqrt{\frac{3}{5}})+\frac{8}{9}.f(0)+\frac{5}{9}.f(\sqrt{\frac{3}{5}})$$

The above equation is called as Gauss-Legendre Three Point Formula.

Gauss-Legendre Three Point Formula ..Contd

To find the integration by Gauss-Legendre Three Point Formula use following steps

1) Write the given function in the form of $I = \int_{x_0}^{x_n} f(x)dx$
2) Calculate the values of 'C' and 'd' by using relation $C = \frac{x_n - x_0}{2}$ & $d = \frac{x_n + x_0}{2}$
3) Calculate the three values of x i.e. x_1, x_2 and x_3 and corresponding values of y by using relation,

$$x_1 = C\left(\sqrt{\frac{3}{5}}\right) + d \ , \ x_2 = -C\left(\sqrt{\frac{3}{5}}\right) + d \text{ and } x_3 = C(0) + d = d$$

$$y_1 = f(x_1)\ , \quad y_2 = f(x_2) \text{ and } y_3 = f(x_3)$$

4) Calculate the area under the curve by using

$$A = \left[\frac{5}{9}(y_1 + y_2) + \frac{8}{9}(y_3)\right].C$$

Gauss-Legendre Three Point Formula ..Contd

Example: Find integration of $e^x cosx - 2x$ in limits 0 to 1 by using Three Point Gauss-Legendre formula.

Solution: $\mathrm{f(x)} = (e^x cosx - 2x)$, x_0=0, x_n=1

Step 1: Calculate the value of C and d

$$C = \frac{x_n - x_0}{2} = \frac{1-(0)}{2} = 0.5$$

$$d = \frac{x_n + x_0}{2} = \frac{1+(0)}{2} = 0.5$$

Step 2: Calculate the value of x_1, x_2 and x_3

$$x_1 = C\left(\sqrt{\frac{3}{5}}\right) + d = 0.5\left(\sqrt{\frac{3}{5}}\right) + 0.5 = 0.8872$$

$$x_2 = -C\left(\sqrt{\frac{3}{5}}\right) + d = -0.5\left(\sqrt{\frac{3}{5}}\right) + 0.5 = 0.1127$$

$$x_3 = d = 0.5$$

Gauss-Legendre Three Point Formula ..Contd

Step 3: Calculate the value of $\boldsymbol{y}, \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = (\boldsymbol{e}^{\boldsymbol{x}}\boldsymbol{cosx} - \boldsymbol{2x})$

$$y_1 = (e^{x_1}cosx_1 - 2x_1) = (e^{0.8872}cos0.8872 - 2(0.8872)) = -0.2407$$
$$y_2 = (e^{x_2}cosx_2 - 2x_2) = (e^{0.1127}cos0.1127 - 2(0.1127)) = 0.8867$$
$$y_3 = (e^{x_3}cosx_3 - 2x_3) = (e^{0.5}cos0.5 - 2(0.5)) = 0.4468$$

Step 4: Calculate the area under the curve (A)

$$A = \left[\frac{5}{9}(y_1 + y_2) + \frac{8}{9}(y_3)\right].C$$
$$= \left[\frac{5}{9}(-0.2407 + 0.8867\,) + \frac{8}{9}(0.4468)\right].0.5$$

$$\mathrm{A} = \int_0^1 (e^x cosx - 2x)dx = 0.37802$$

Problems on Gauss-Legendre Three Point formula

1. Using Gauss-Legendre Three Point formula $\int_3^5 (x^2 - 5x + 2)dx$.
2. Find the integration $\int_0^2 (e^x + 4x - 3)dx$ using Gauss-Legendre Three Point formula.
3. Evaluate $\int_0^1 (\frac{1}{1+x^2})dx$ using Gauss-Legendre Three Point formula.
4. Find the integration $\int_0^4 (x^3 - cosx + 6)dx$ using Gauss-Legendre Three Point formula.

Problems on Gauss-Legendre Three Point formula .....Contd

5. Compute the integral $\int_0^{\pi/2}(e^{sinx})dx$ using Gauss-Legendre Three Point formula.
6. Use Three Point Gauss-Legendre formula to solve $\int_0^3(\frac{e^x}{1+x^2})dx$.
7. A fluid is confined in a cylinder by a spring loaded frictionless piston so that the pressure in the fluid is a linear function of volume P=a+bV where P is in Kpa, V is in m^3, a=-60 kN/m^2, b=7667 kN/m^2. If the fluid changes from initial condition of 0.03 m^3 to final volume of 0.06 m^3. Find the magnitude of work transfer during the process using Gauss-Legendre 3 point formula.

Reference Books

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Numerical Double Integration (Simpson's 1/3rd Rule)

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Double Integration

The double integration can be written as

$$A = \int_{x_0}^{x_n} \int_{y_0}^{y_n} f(x,y)dx\,dy$$

The value of integration can be found by two successive integration in x and y directions (by considering one variable at a time). In this case, the interval x_0 to x_n is divided into *'n'* equal subintervals with step size *'h'* whereas the interval y_0 to y_n is divided into *'m'* equal subintervals with step size *'k'*.

$$h = \frac{x_n - x_0}{n} \text{ and } k = \frac{y_n - y_0}{m}$$

The double integration can be found by using (1) Trapezoidal Rule (2) Simpson's Rule

Simpson's 1/3rd Rule

Double integration is given by

$$A = \int_{x_0}^{x_n} \int_{y_0}^{y_n} f(x,y)dx\,dy$$

In the given equation if *n*=*m*=2 then,

$$A = \int_{x_0}^{x_2} \int_{y_0}^{y_2} f(x,y)dx\,dy$$

Initially integrating the above equation *w.r.t.* x

$$A = \int_{y_0}^{y_2} \frac{h}{3}[f(x_0,y) + 4f(x_1,y) + +f(x_2,y)]\,dy$$

Now integrating the above equation *w.r.t.* y

$$A = \frac{h}{3}\frac{k}{3}[f(x_0,y_0) + 4f(x_0,y_1) + f(x_0,y_2) + 4[f(x_1,y_0) + 4f(x_1,y_1) + f(x_1,y_2)] + f(x_2,y_0) + 4f(x_2,y_1) + f(x_2,y_2)]$$

Simpson's 1/3rd Rule ...Contd

$$A = \frac{h}{3}\frac{k}{3}[f(x_0,y_0) + f(x_0,y_2)) + f(x_2,y_0) + f(x_2,y_2) + 4[f(x_0,y_1) + f(x_1,y_0) + f(x_1,y_2) + f(x_2,y_1)] + 16f(x_1,y_1)]$$

When number of strips n=m=2, the table is prepared as follows

y \ x	x_0	$x_1 = x_0 + h$	$x_2 = x_0 + 2h$
y_0	$f(x_0,y_0)$	$f(x_1,y_0)$	$f(x_2,y_0)$
$y_1 = y_0 + k$	$f(x_0,y_1)$	$f(x_1,y_1)$	$f(x_2,y_1)$
$y_2 = y_0 + 2k$	$f(x_0,y_2)$	$f(x_1,y_2)$	$f(x_2,y_2)$

y \ x	x_0	$x_1 = x_0 + h$	$x_2 = x_0 + 2h$
y_0			
$y_1 = y_0 + k$		A_1	
$y_2 = y_0 + 2k$			

Simpson's 1/3rd Rule ...Contd

$$A = \frac{h}{3}\frac{k}{3}\left[\sum f(x_i, y_i) + 4\sum f(x_i, y_i) + 16\sum f(x_i, y_i)\right]$$

Square terms Rhombus terms Remaining terms

For *n* number of strips prepare a table as follows

y \ X	x_0	$x_1 = x_0 + h$	$x_2 = x_0 + 2h$	$x_3 = x_0 + 3h$	$x_4 = x_0 + 4h$
y_0	$f(x_0, y_0)$	$f(x_1, y_0)$	$f(x_2, y_0)$	$f(x_3, y_0)$	$f(x_4, y_0)$
$y_1 = y_0 + k$	$f(x_0, y_1)$	A_1 $f(x_1, y_1)$	$f(x_2, y_1)$	A_2 $f(x_3, y_1)$	$f(x_4, y_1)$
$y_2 = y_0 + 2k$	$f(x_0, y_2)$	$f(x_1, y_2)$	$f(x_2, y_2)$	$f(x_3, y_2)$	$f(x_4, y_2)$
$y_3 = y_0 + 3k$	$f(x_0, y_3)$	A_3 $f(x_1, y_3)$	$f(x_2, y_3)$	A_4 $f(x_3, y_3)$	$f(x_4, y_3)$
$y_4 = y_0 + 4k$	$f(x_0, y_4)$	$f(x_1, y_4)$	$f(x_2, y_4)$	$f(x_3, y_4)$	$f(x_4, y_4)$

Simpson's 1/3rd Rule ...Contd

The total area is given by

$$A = \frac{h}{3}\frac{k}{3}[A_1 + A_2 + A_3 + A_4]$$

Where

$$A_1 = \left[\left(f(x_0,y_0) + f(x_2,y_0) + f(x_2,y_2) + f(x_0,y_2)\right) + 4\left(f(x_1,y_0) + f(x_2,y_1) + f(x_1,y_2) + f(x_0,y_1)\right) + 16f(x_1,y_1)\right]$$

$$A_2 = \left[\left(f(x_2,y_0) + f(x_4,y_0) + f(x_4,y_2) + f(x_2,y_2)\right) + 4\left(f(x_3,y_0) + f(x_4,y_1) + f(x_3,y_2) + f(x_2,y_1)\right) + 16f(x_3,y_1)\right]$$

Note:- While calculating area A_2, the terms in the 3rd column are repeated.

Simpson's 1/3rd Rule ...Contd

$$A_3 = \left[\left(f(x_0,y_2)+f(x_2,y_2)+f(x_2,y_4)+f(x_0,y_4)\right) + 4\left(f(x_1,y_2)+f(x_2,y_3)+f(x_1,y_4)+f(x_0,y_3)\right)+16f(x_1,y_3)\right]$$

Note:- While calculating area A_3 the term in the 3rd row are repeated.

$$A_4 = \left[\left(f(x_2,y_2)+f(x_4,y_2)+f(x_4,y_4)+f(x_2,y_4)\right) + 4\left(f(x_3,y_2)+f(x_4,y_3)+f(x_3,y_4)+f(x_2,y_3)\right)+16f(x_3,y_3)\right]$$

Note:- While calculating area A_4 the term in the 3rd row and 3rd column are repeated.

The above equation gives the total area under the curve for double integration by Simpson's 1/3rd rule.

Simpson's 1/3rd Rule ...Contd

Example: Find the integral of f(x,y) =(x^2+y^2+5) for x=0 to 2 and y=0 to 2 taking increment in both x and y as 0.5 applying Simpson's 1/3rd rule.

Solution: $\mathrm{f}(\mathrm{x}, y) = (x^2+y^2+5)$, x_0=0, x_n=2, y_0=0, y_m=2, h=k=0.5

Step 1: Calculate the corresponding values of x *and y wrt* h & k

$$x_0 = 0$$

$$x_1 = x_0 + \mathrm{h} = 0 + 0.5 = 0.5$$

$$x_2 = x_0 + 2\mathrm{h} = 0 + 2(0.5) = 1.0$$

$$x_3 = x_0 + 3\mathrm{h} = 0 + 3(0.5) = 1.5$$

$$x_4 = x_0 + 4\mathrm{h} = 0 + 4(0.5) = 2.0$$

Simpson's 1/3rd Rule ...Contd

$$y_0 = 0$$
$$y_1 = y_0 + \mathrm{k} = 0 + 0.5 = 0.5$$
$$y_2 = y_0 + 2\mathrm{k} = 0 + 2(0.5) = 1.0$$
$$y_3 = y_0 + 3\mathrm{k} = 0 + 3(0.5) = 1.5$$
$$y_4 = y_0 + 4\mathrm{k} = 0 + 4(0.5) = 2.0$$

Step 2: Calculate the values of $f(x_i, y_i)$, $\mathbf{f}(\mathbf{x}, y) = (x^2 + y^2 + 5)$

1) x_0=constant

$$f(x_0, y_0) = ({x_0}^2 + {y_0}^2 + 5) = (0 + 0 + 5) = 5$$
$$f(x_0, y_1) = ({x_0}^2 + {y_1}^2 + 5) = (0 + 0.5^2 + 5) = 5.25$$
$$f(x_0, y_2) = ({x_0}^2 + {y_2}^2 + 5) = (0 + 1^2 + 5) = 6$$
$$f(x_0, y_3) = ({x_0}^2 + {y_3}^2 + 5) = (0 + 1.5^2 + 5) = 7.25$$
$$f(x_0, y_4) = ({x_0}^2 + {y_4}^2 + 5) = (0 + 2^2 + 5) = 9$$

Simpson's 1/3rd Rule ...Contd

2) x_1=constant

$$f(x_1, y_0)=({x_1}^2+{y_0}^2+5) = (0.5^2 + 0^2 + 5) = 5.25$$
$$f(x_1, y_1)=({x_1}^2+{y_1}^2+5) = (0.5^2 + 0.5^2 + 5) = 5.5$$
$$f(x_1, y_2)=({x_1}^2+{y_2}^2+5) = (0.5^2 + 1^2 + 5) = 6.25$$
$$f(x_1, y_3)=({x_1}^2+{y_3}^2+5) = (0.5^2 + 1.5^2 + 5) = 7.5$$
$$f(x_1, y_4)=({x_1}^2+{y_4}^2+5) = (0.5^2 + 2^2 + 5) = 9.25$$

3) x_2=constant

$$f(x_2, y_0)=({x_2}^2+{y_0}^2+5) = (1^2 + 0^2 + 5) = 6$$
$$f(x_2, y_1)=({x_2}^2+{y_1}^2+5) = (1^2 + 0.5^2 + 5) = 6.25$$
$$f(x_2, y_2)=({x_2}^2+{y_2}^2+5) = (1^2 + 1^2 + 5) = 7$$
$$f(x_2, y_3)=({x_2}^2+{y_3}^2+5) = (1^2 + 1.5^2 + 5) = 8.25$$
$$f(x_2, y_4)=({x_2}^2+{y_4}^2+5) = (1^2 + 2^2 + 5) = 10$$

Simpson's 1/3rd Rule ...Contd

4) x_3=constant

$$f(x_3, y_0) = ({x_3}^2 + {y_0}^2 + 5) = (1.5^2 + 0^2 + 5) = 7.25$$
$$f(x_3, y_1) = ({x_3}^2 + {y_1}^2 + 5) = (1.5^2 + 0.5^2 + 5) = 7.5$$
$$f(x_3, y_2) = ({x_3}^2 + {y_2}^2 + 5) = (1.5^2 + 1^2 + 5) = 8.25$$
$$f(x_3, y_3) = ({x_3}^2 + {y_3}^2 + 5) = (1.5^2 + 1.5^2 + 5) = 9.5$$
$$f(x_3, y_4) = ({x_3}^2 + {y_4}^2 + 5) = (1.5^2 + 2^2 + 5) = 11.25$$

5) x_4=constant

$$f(x_4, y_0) = ({x_4}^2 + {y_0}^2 + 5) = (2^2 + 0^2 + 5) = 9$$
$$f(x_4, y_1) = ({x_4}^2 + {y_1}^2 + 5) = (2^2 + 0.5^2 + 5) = 9.25$$
$$f(x_4, y_2) = ({x_4}^2 + {y_2}^2 + 5) = (2^2 + 1^2 + 5) = 10$$
$$f(x_4, y_3) = ({x_4}^2 + {y_3}^2 + 5) = (2^2 + 1.5^2 + 5) = 11.25$$
$$f(x_4, y_4) = ({x_4}^2 + {y_4}^2 + 5) = (2^2 + 2^2 + 5) = 13$$

Simpson's 1/3rd Rule ...Contd

Step 3: Make a table for all values of f(x,y)

y \ X	x_0	$x_1 = x_0 + h$	$x_2 = x_0 + 2h$	$x_3 = x_0 + 3h$	$x_4 = x_0 + 4h$
y_0	$f(x_0, y_0)$	$f(x_1, y_0)$	$f(x_2, y_0)$	$f(x_3, y_0)$	$f(x_4, y_0)$
$y_1 = y_0 + k$	$f(x_0, y_1)$	A_1 $f(x_1, y_1)$	$f(x_2, y_1)$	A_2 $f(x_3, y_1)$	$f(x_4, y_1)$
$y_2 = y_0 + 2k$	$f(x_0, y_2)$	$f(x_1, y_2)$	$f(x_2, y_2)$	$f(x_3, y_2)$	$f(x_4, y_2)$
$y_3 = y_0 + 3k$	$f(x_0, y_3)$	A_3 $f(x_1, y_3)$	$f(x_2, y_3)$	A_4 $f(x_3, y_3)$	$f(x_4, y_3)$
$y_4 = y_0 + 4k$	$f(x_0, y_4)$	$f(x_1, y_4)$	$f(x_2, y_4)$	$f(x_3, y_4)$	$f(x_4, y_4)$

Simpson's 1/3rd Rule ...Contd

y \ x	0	0.5	1	1.5	2
0	5	5.25	6	7.25	9
0.5	5.25	A_1 5.5	6.25	A_2 7.5	9.25
1	6	6.25	7	8.25	10
1.5	7.25	A_3 7.5	8.25	A_4 9.5	11.25
2	9	9.25	10	11.25	13

Simpson's 1/3rd Rule ...Contd

Step 4: Calculate the area under the curve by Simpson's 1/3rd rule

As per Simpson's 1/3rd rule area A_1 is given by

$$A_1 = \left[\sum f(x_i, y_i) + 4\sum f(x_i, y_i) + 16\sum f(x_i, y_i)\right]$$

Square terms Rhombus terms Remaining terms

$$A_1 = \left[\left(f(x_0, y_0) + f(x_2, y_0) + f(x_2, y_2) + f(x_0, y_2)\right) + 4\left(f(x_1, y_0) + f(x_2, y_1) + f(x_1, y_2) + f(x_0, y_1)\right) + 16f(x_1, y_1)\right]$$

$$A_1 = [(5 + 6 + 7 + 6) + 4(5.25 + 6.25 + 6.25 + 5.25) + 16(5.5)]$$

$$\boldsymbol{A_1 = 204}$$

Simpson's 1/3rd Rule ...Contd

For calculating area A_2, the terms in the 3rd column are repeated

$$A_2 = \left[\left(f(x_2,y_0)+f(x_4,y_0)+f(x_4,y_2)+f(x_2,y_2)\right) + 4\left(f(x_3,y_0)+f(x_4,y_1)+f(x_3,y_2)+f(x_2,y_1)\right) + 16f(x_3,y_1)\right]$$

$$A_2 = [(6+9+10+7)+4(7.25+9.25+8.25+6.25)+16(7.5)]$$

$$\boldsymbol{A_2 = 276}$$

Simpson's 1/3rd Rule ...Contd

For calculating area A_3, the terms in the 3rd row are repeated

$$A_3 = \left[\left(f(x_0,y_2) + f(x_2,y_2) + f(x_2,y_4) + f(x_0,y_4)\right) + 4\left(f(x_1,y_2) + f(x_2,y_3) + f(x_1,y_4) + f(x_0,y_3)\right) + 16f(x_1,y_3)\right]$$

$$A_3 = [(6 + 7 + 10 + 9) + 4(6.25 + 8.25 + 9.25 + 7.25) + 16(7.5)]$$

$$\boldsymbol{A_3 = 276}$$

Simpson's 1/3rd Rule ...Contd

For calculating area A_4, the terms in the 3rd row and 3rd column are repeated

$$A_4 = \left[\left(f(x_2, y_2) + f(x_4, y_2) + f(x_4, y_4) + f(x_2, y_4)\right) + 4\left(f(x_3, y_2) + f(x_4, y_3) + f(x_3, y_4) + f(x_2, y_3)\right) + 16 f(x_3, y_3)\right]$$

$$A_4 = [(7 + 10 + 13 + 10) + 4(8.25 + 11.25 + 11.25 + 8.25) + 16(9.5)]$$

$$\boldsymbol{A_4 = 348}$$

Simpson's 1/3rd Rule ...Contd

The total area is given by

$$A = \frac{h}{3}\frac{k}{3}[A_1 + A_2 + A_3 + A_4]$$

$$A = \frac{0.5}{3}\frac{0.5}{3}[204 + 276 + 276 + 348]$$

$$A = 30.6666$$

Problems on Simpson's 1/3rd Rule

1. Find $\int_0^1 \int_0^1 (e^{x+2y}) dxdy$ using Simpson's 1/3rd rule take h=k=0.5.
2. Find double integral of $\mathrm{f(x,y)}$ =2x+y+1 for x=0 to 2 and y=0 to 2 with step size for both x and y as 1 by using Simpson's 1/3rd rule .
3. Evaluate $\int_6^{14} \int_1^5 \left(\frac{x+xy}{2y}\right) dxdy$ by Simpson's 1/3rd rule. Take number of strips for x and y equal to 4.
4. Evaluate $\int_6^{14} \int_1^5 (x - y + 1)\, dxdy$ using Simpson's 1/3rd rule with number of strips for x and y equal to 4.

Reference Books

1. Steven C. Chapra, Raymond P. Canale, Numerical Methods for Engineers, 4/e, Tata McGraw Hill Editions

2. Dr. B. S. Garewal, Numerical Methods in Engineering and Science, Khanna Publishers,.

3. Steven C. Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientist, Tata Mc-Graw Hill Publishing Co-Ltd

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5. Gerald and Wheatley, Applied Numerical Analysis, Pearson Education Asia

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8. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Thank You

Numerical Integration (Simpson's 1/3rd Rule)

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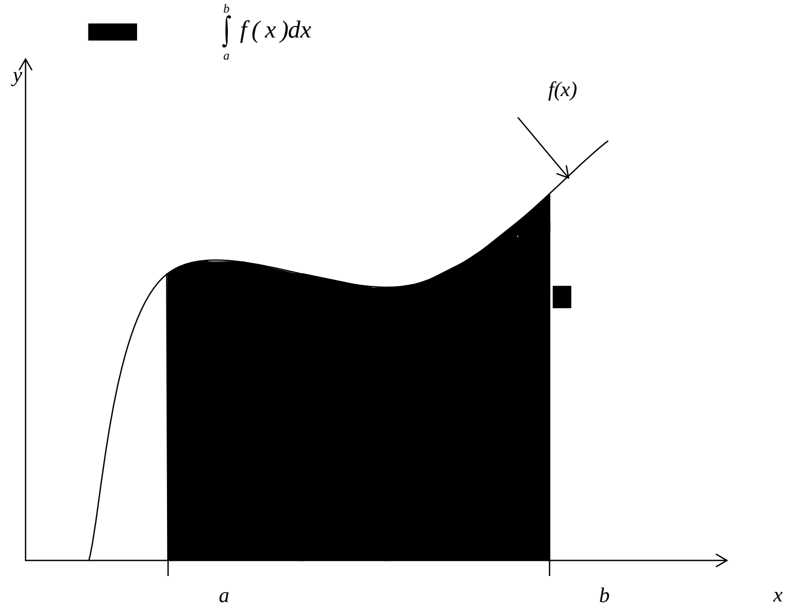
What is Integration ?

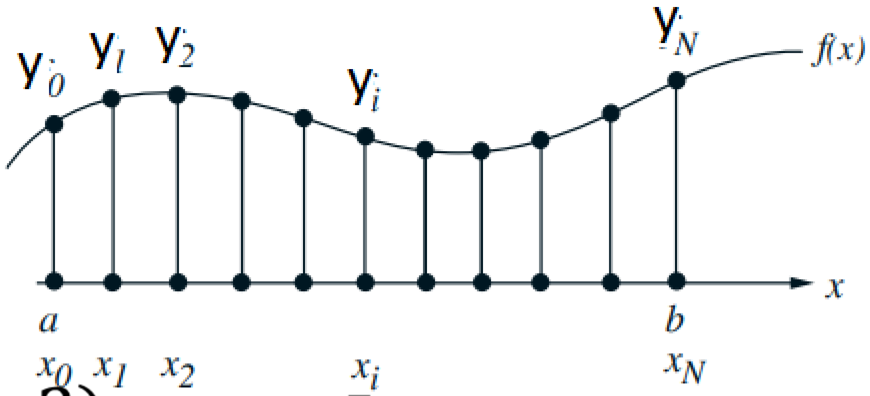
- The process of measuring the area under a function plotted on a graph.

$$I = \int_a^b f(x)dx$$



- Where:
- *f(x)* is the integrand
- a= lower limit of integration
- b= upper limit of integration

Newton-Cotes Quadrature Formula

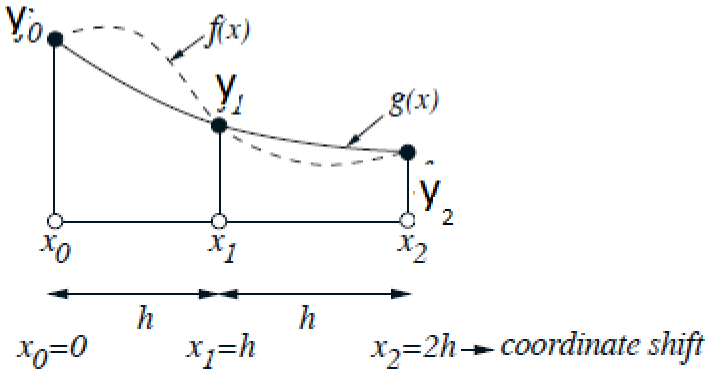


$$I = nh\left[\begin{array}{l} y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)}{24}\Delta^3 y_0 + \\ \left(\frac{n^4}{5} - \frac{3n^3}{2} + \frac{11n^2}{3} - 3n\right)\frac{\Delta^4 y_0}{24} \ldots\ldots\ldots\ldots\ldots\ldots.. \end{array}\right]$$

- Where
- $\Delta y_0 = y_1 - y_0, \Delta^2 y_0 = y_2 - 2y_1 + y_0, \Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$

Simpson's 1/3rd Rule

The curve which bounds each strip is approximated as a parabola (second degree polynomial). The Newton-Cotes formula for n=2 becomes



$$I = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0\right]$$

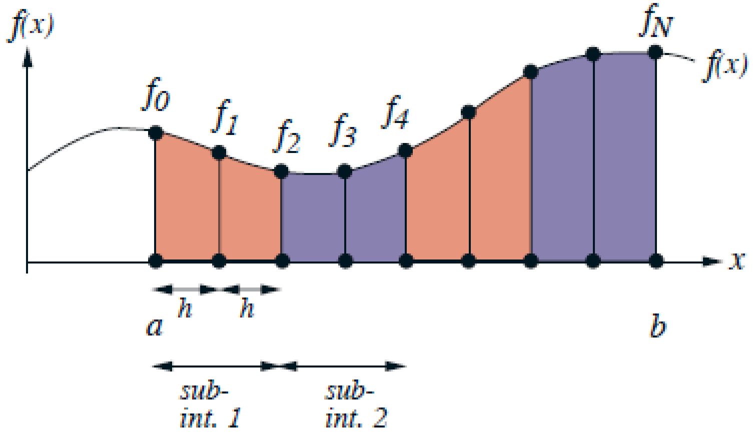
Area under the curve is

$$A = 2h\left[y_0 + \frac{2}{2}(y_1 - y_0) + \frac{2(2x2-3)}{12}(y_2 - 2y_1 + y_0)\right]$$

$$A = \frac{h}{3}[y_0 + 4y_1 + y_2]$$

Simpson's 1/3rd Rule ...Contd

Consider a curve as shown in figure which is divided in n number of strips and joined by a parabola.



Let $A_1, A_2, A_3, \ldots. A_n$ be the area under each strip

Area under 1st strip, $A_1 = \frac{h}{3}[y_0 + 4y_1 + y_2]$

Area under 2nd strip, $A_2 = \frac{h}{3}[y_2 + 4y_3 + y_4]$

Simpson's 1/3rd Rule ...Contd

Area under nth strip, $A_n = \frac{h}{3}[y_{n-2} + 4y_{n-1} + y_n]$

- Total area under the curve is A = $A_1 + A_2 + A_3 + ... + A_n$

A=$\frac{h}{3}[y_0 + 4y_1 + y_2]$+$\frac{h}{3}[y_2 + 4y_3 + y_4]$+ ....+$\frac{h}{3}[y_{n-2} + 4y_{n-1} + y_n]$

A= $\frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots\ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots\ldots + y_{n-2})]$

Simpson's 1/3rd Rule ...Contd

A= $\frac{h}{3}[(y_0 + y_n) + 4(odd\ terms\ of\ y) + 2(even\ terms\ of\ y)]$

Note: For using Simpsons 1/3rd Rule, the number of strips (n) should be multiple of two.

The above equation is used to calculate area under the curve by Simpsons 1/3rd Rule.

Simpson's 1/3rd Rule ...Contd

Example: Use Simpson's 1/3rd rule to estimate integration $\int_1^2 (\frac{e^x}{x}) dx$.

Solution: $\mathrm{f(x)} = (\frac{e^x}{x})$, initial limit x_0=1, final limit x_n=2, Let n=6

$$h = \left(\frac{x_n - x_0}{n}\right) = \left(\frac{2-1}{6}\right) = 0.1667$$

Step 1: Calculate the corresponding values of x *wrt* h

$x_0 = 1$

$x_1 = x_0 + \mathrm{h} = 1 + 0.1667 = 1.1667$

$x_2 = x_0 + 2\mathrm{h} = 1 + 2\mathrm{x}\ 0.1667 = 1.3333$

$x_3 = x_0 + 3\mathrm{h} = 1 + 3\mathrm{x}\ 0.1667 = 1.5$

Simpson's 1/3rd Rule ...Contd

$x_4 = x_0 + 4h$= 1 + 4x 0.1667 = 1.6667

$x_5 = x_0 + 5h$= 1 + 5x 0.1667 = 1.8333

$x_6 = x_0 + 6h$= 1 + 6x 0.1667 = 2

Step 2: Calculate the corresponding values of y *wrt* x

$$y_0 = \frac{e^{x_0}}{x_0} = \frac{e^1}{1} = 2.7182$$

$$y_1 = \frac{e^{x_1}}{x_1} = \frac{e^{1.1667}}{1.1667} = 2.7525$$

$$y_2 = \frac{e^{x_2}}{x_2} = \frac{e^{1.3333}}{1.3333} = 2.8452$$

$$y_3 = \frac{e^{x_3}}{x_3} = \frac{e^{1.5}}{1.5} = 2.9877$$

Simpson's 1/3rd Rule ...Contd

$$y_4 = \frac{e^{x_4}}{x_4} = \frac{e^{1.6667}}{1.6667} = 3.1767$$

$$y_5 = \frac{e^{x_5}}{x_5} = \frac{e^{1.8333}}{1.8333} = 3.4116$$

$$y_6 = \frac{e^{x_6}}{x_6} = \frac{e^{2}}{2} = 3.6945$$

Step 3: Calculate the area under the curve by Simpson's 1/3rd rule

A= $\frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$

A= $\frac{0.1667}{3}[(2.7182 + 3.6945) + 4(2.7525 + 2.9877 + 3.4116) + 2(2.8452 + 3.1767)]$

$$\int_1^2 \left(\frac{e^x}{x}\right)dx = 3.0597$$

Problems on Simpson's 1/3rd Rule

1. Evaluate $\int_0^{0.8}(log_e(x+1)+\sin(2x))dx$ where x is in radians using Simpson's 1/3rd rule by dividing entire interval in 8 strips.
2. The velocity of car running on a straight road at the interval of 2 minutes is given below. Find the distance covered by the car using Simpson's 1/3rd rule.

Time(min)	0	2	4	6	8	10	12
Velocity (km/hr)	0	22	30	27	18	7	0

3. The velocity V (km/hr) of a vehicle which starts from rest is given at fixed interval of time 't' (min) as follows. Estimate approximately the distance covered in 20 minutes.

Time(min)	2	4	6	8	10	12	14	16	18	20
V (km/hr)	10	18	25	29	32	20	11	5	2	0

Problems on Simpson's 1/3rd Rule ...Contd

4. A circular shaft having one meter length has varying radius 'r' as follows. An axial pule of 300 KN is applied at one end of the shaft whose modulus of elasticity is 200×10^9 N/m^2. The axial elongation of the shaft (Δx) is given by $\Delta x = (P/_E)\int_0^1 (1/_A)dx$ where A=cross sectional area of shaft. Determine elongation of shaft over the entire length by Simpson's 1/3rd rule.

x (m)	0	0.25	0.50	0.75	1.00
r (m)	1.00	0.9896	0.9589	0.9089	0.8415

5. Gas is expanded according to law $PV^{1.3} = C$ from the pressure of 10 N/m2. Assuming initial volume of gas as 1 m^3 and final volume as 7 m^3. Calculate work done using Simpson's 1/3rd rule. Divide volume in 6 equal strips.

6. The data listed in table gives measurements of heat flux q at the surface of a solar collector. Estimate the total heat absorbed by a 2×10^5 cm^2 collector panel during 14 hours period. The panel has an absorption efficiency ε=42%. The total heat absorbed is given by $H = \varepsilon \int_0^t q.A.dt$ where A is area, q is heat flux and t is time.

t (hr)	0	1	2	3	4	6	8	11	14
q (N/cm^{2}hr	0.05	1.72	5.23	6.38	7.86	8.05	8.03	5.82	0.24

Problems on Simpson's 1/3rd Rule ...Contd

7. Find out $\int_1^{2.2} y.dx$ using following table by Simpson's 1/3rd rule.

x	1	1.1	1.2	1.4	1.6	1.9	2.2
y	3	4.2	5.8	10	14.5	25.1	40

8. Evaluate $\int_0^4 e^x dx$ using Simpson's 1/3rd rule for four strips.
9. Evaluate $\int_1^4 (e^x - x^3 - 2x + 1)dx$ using Simpson's 1/3rd rule taking 6 divisions.
10. Evaluate $\int_0^1 \left(\frac{sinx}{x}\right) dx$ using Simpson's 1/3rd rule with h=1/6.

Problems on Simpson's 1/3rd Rule ...Contd

11. Evaluate $\int_0^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4)\, dx$ by using Simpson's 1/3rd rule using 4 intervals.

Reference Books

1. Steven C. Chapra, Raymond P. Canale, Numerical Methods for Engineers, 4/e, Tata McGraw Hill Editions

2. Dr. B. S. Garewal, Numerical Methods in Engineering and Science, Khanna Publishers,.

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Reference Books ...Contd

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8. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Thank You

Numerical Integration (Simpson's 3/8th Rule)

Compiled by

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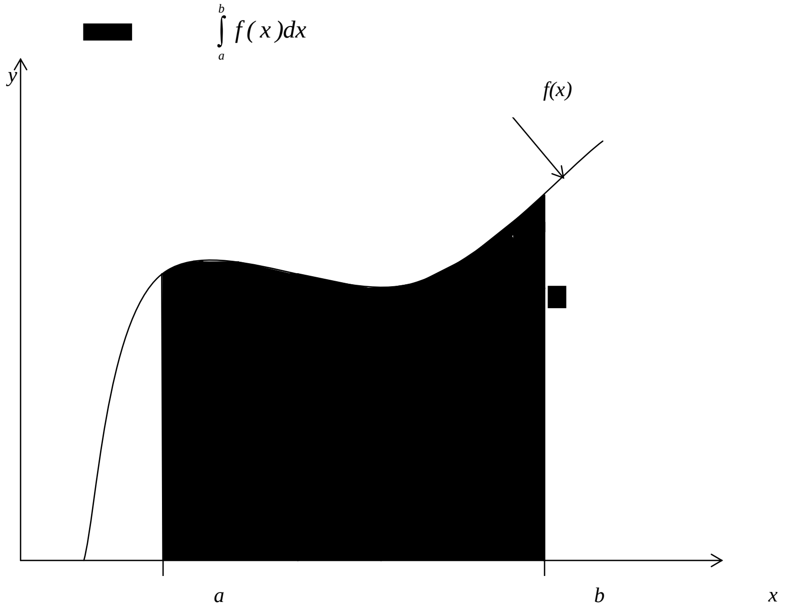
What is Integration ?

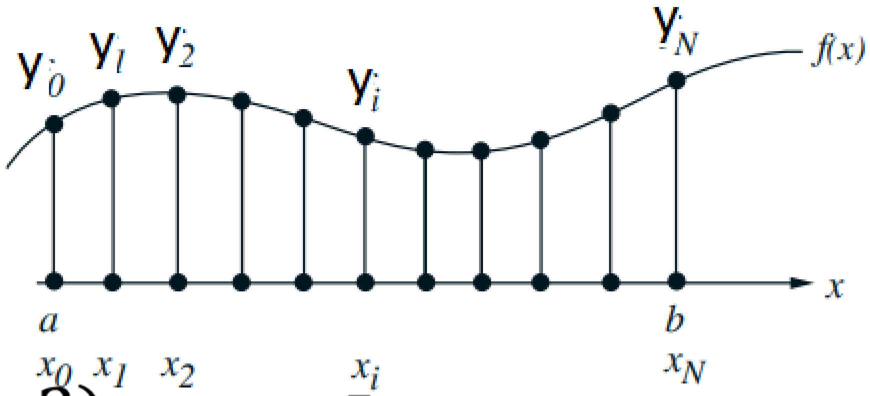
- The process of measuring the area under a function plotted on a graph.

$$I = \int_a^b f(x)dx$$

- Where:
- *f(x)* is the integrand
- a= lower limit of integration
- b= upper limit of integration



Newton-Cotes Quadrature Formula



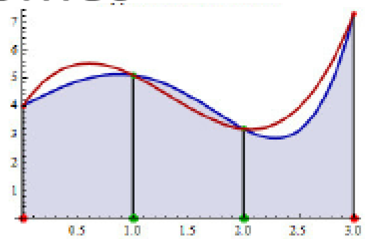
$$I = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)}{24}\Delta^3 y_0 + \left(\frac{n^4}{5} - \frac{3n^3}{2} + \frac{11n^2}{3} - 3n\right)\frac{\Delta^4 y_0}{24} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \right]$$

- Where
- $\Delta y_0 = y_1 - y_0, \Delta^2 y_0 = y_2 - 2y_1 + y_0, \Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$

Simpson's 3/8th Rule

The curve which bounds each strip is approximated as a cubic curve (third degree polynomial). The Newton-Cotes formula for n=3 becomes

$$I = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)}{24}\Delta^3 y_0\right]$$



Area under the curve is AA

$$= 3h\left[y_0 + \frac{3}{2}(y_1 - y_0) + \frac{3(2x3-3)}{12}(y_2 - 2y_1 + y_0) + \frac{3(3-2)}{24}(y_3 - 3y_2 + 3y_1 - y_0)\right]$$

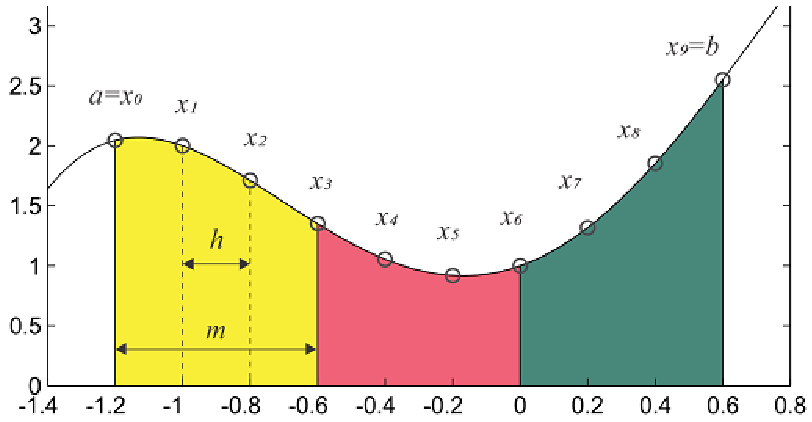
$$A = \frac{3h}{8}[y_0 + 3y_1 + 3y_2 + y_3]$$

Simpson's 3/8th Rule ...Contd

Consider a curve as shown in figure which is divided in n number of strips and joined by a cubic curve.

Let A_1, A_2,A_3,....A_n be the area under

Each strip



Area under 1st strip, $A_1 = \frac{3h}{8}[y_0 + 3y_1 + 3y_2 + y_3]$

Area under 2nd strip, $A_2 = \frac{3h}{8}[y_3 + 3y_4 + 3y_5 + y_6]$

Simpson's 3/8th Rule ...Contd

Area under nth strip, $A_n = \frac{3h}{8}[y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n]$

- Total area under the curve is A = $A_1 + A_2 + A_3 + \ldots. + A_n$

A=$\frac{3h}{8}[y_0 + 3y_1 + 3y_2 + y_3]$+$\frac{3h}{8}[y_3 + 3y_4 + 3y_5 + y_6]$+ ……………….+$\frac{3h}{8}[y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n]$

A= $\frac{3h}{8}[(y_0 + y_n) + 2(y_3 + y_6 + \ldots\ldots +2y_{n-3}) + 3(y_1 + y_2 + y_4 \ldots\ldots +y_{n-2} +y_{n-1})]$

Simpson's 3/8th Rule …Contd

A= $\frac{3h}{8}[(y_0 + y_n) + 2(y\ terms\ which\ are\ multiple\ of\ 3) + 3(remaining\ terms\ of\ y)]$

Note: For using Simpsons 3/8th Rule, the number of strips (n) should be multiple of three.

The above equation is used to calculate area under the curve by Simpsons 3/8th Rule.

Simpson's 3/8th Rule ...Contd

Example: Evaluate $\int_0^1 (\frac{sinx}{2+3sinx})dx$ using Simpson's 3/8th Rule take 6 strips.

Solution: $f(x) = (\frac{sinx}{2+3sinx})$, initial limit x_0=0, final limit x_n=1, n=6

$$h = \left(\frac{x_n - x_0}{n}\right) = \left(\frac{1-0}{6}\right) = \frac{1}{6}$$

Note:- Keep the calculator in radian mode since trignometric function

Step 1: Calculate the corresponding values of x *wrt* h

x_0 = 0

$x_1 = x_0 + \mathrm{h}$ = 0 + 1/6 = 1/6

$x_2 = x_0 + 2\mathrm{h}$= 0 + 2(1/6) = 1/3

$x_3 = x_0 + 3\mathrm{h}$= 0 + 3(1/6) = 1/2

Simpson's 3/8th Rule ...Contd

$x_4 = x_0 + 4h = 0 + 4(1/6) = 2/3$

$x_5 = x_0 + 5h = 0 + 5(1/6) = 5/6$

$x_6 = x_0 + 6h = 0 + 6(1/6) = 1$

Step 2: Calculate the corresponding values of y *wrt* x

$$y_0 = \frac{sinx_0}{2 + 3sinx_0} = \frac{sin0}{2 + 3sin0} = 0$$

$$y_1 = \frac{sinx_1}{2+3sinx_1} = \frac{\sin(\frac{1}{6})}{2+3\sin(\frac{1}{6})} = 0.06641$$

$$y_2 = \frac{sinx_2}{2 + 3sinx_2} = \frac{\sin(\frac{1}{3})}{2 + 3\sin(\frac{1}{3})} = 0.1097$$

$$y_3 = \frac{sinx_3}{2 + 3sinx_3} = \frac{\sin(\frac{1}{2})}{2 + 3\sin(\frac{1}{2})} = 0.1394$$

Simpson's 3/8th Rule ...Contd

$$y_4 = \frac{sinx_4}{2 + 3sinx_4} = \frac{\sin(\frac{2}{3})}{2 + 3\sin(\frac{2}{3})} = 0.1604$$

$$y_5 = \frac{sinx_5}{2 + 3sinx_5} = \frac{\sin(\frac{5}{6})}{2 + 3\sin(\frac{5}{6})} = 0.1753$$

$$y_6 = \frac{sinx_6}{2 + 3sinx_6} = \frac{\sin(1)}{2 + 3\sin(1)} = 0.1859$$

<u>Step 3: Calculate the area under the curve by Simpson's 3/8th Rule</u>

A= $\frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$

A= $\frac{3(\frac{1}{6})}{8}[(0 + 0.1859\ ) + 3(0.06641\ \ + 0.1097\ \ + 0.1604\ \ + 0.1753\ ) + 2(0.1394\ )]$

$\int_0^1 (\frac{sinx}{2+3sinx})dx$=0.1250

Problems on Simpson's 3/8th Rule

1. Evaluate $\int_1^4 (4x - 1)dx$ using Simpson's 3/8th rule using 6 strips.
2. Find the integration of $\int_0^{\pi} (4 + 2sinx)dx$ using Simpson's 3/8th rule using 6 strips.
3. Find the integration of $\int_0^{1.5} (x^3 - 3x^2 + 6x + 8)dx$ using Simpson's 3/8th rule using 3 strips.
4. Find the integration of $\int_{\pi}^{4\pi} \left(\frac{sinx.log_e x}{cosx}\right) dx$ using Simpson's 3/8th rule using 3 strips.

Problems on Simpson's 3/8th Rule ...Contd

5. Evaluate $\int_0^{\pi} \left(\frac{\sin^2 x}{e^x + \cos x}\right) dx$ using Simpson's 3/8th rule take 6 strips.
6. A body is in the form of a solid of revolution. The diameter D in cms of its section at distance x cm from one end is given below. Estimate volume of the solid.

x	0	2.5	5.0	7.5	10.0	12.5	15.0
D	5	5.5	6.0	6.75	6.25	5.5	4.0

7. The table below shows temperature as function of time. use Simpson's 3/8th rule to estimate $\int_1^7 f(t)dt$.

t	1	2	3	4	5	6	7
T	81	75	80	83	78	70	60

Problems on Simpson's 3/8th Rule ...Contd

8. Using the following data calculate the work done by stretching the spring that has a spring constant of K=300 N/m from x=0 to x=0.3 m. Use Simpson's 1/3rd and 3/8th rule.

F(10^3 N)	0	0.01	0.028	0.046	0.063	0.082	0.11
x, m	0	0.05	0.10	0.15	0.20	0.25	0.30

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Thank You

Numerical Double Integration (Trapezoidal Rule)

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Double Integration

The double integration can be written as

$$A = \int_{x_0}^{x_n} \int_{y_0}^{y_n} f(x,y)dx\, dy$$

The value of integration can be found by two successive integration in x and y directions (by considering one variable at a time). In this case, the interval x_0 to x_n is divided into *'n'* equal subintervals with step size *'h'* whereas the interval y_0 to y_n is divided into *'m'* equal subintervals with step size *'k'.*

$$h = \frac{x_n - x_0}{n} \text{ and } k = \frac{y_n - y_0}{m}$$

The double integration can be found by using (1) Trapezoidal Rule (2) Simpson's Rule

Trapezoidal Rule

Double integration is given by

$$A = \int_{x_0}^{x_n} \int_{y_0}^{y_n} f(x, y) dx\, dy$$

In the given equation if *n*=*m*=1 then,

$$A = \int_{x_0}^{x_1} \int_{y_0}^{y_1} f(x, y) dx\, dy$$

Initially integrating the above equation *w.r.t.* x

$$A = \int_{y_0}^{y_1} \frac{h}{2} [f(x_0, y) + f(x_1, y)]\, dy$$

Now integrating the above equation *w.r.t.* y

$$A = \frac{h}{2} \frac{k}{2} [f(x_0, y_0) + f(x_1, y_0) + f(x_0, y_1) + f(x_1, y_1)]$$

X / y	x_0	$x_1 = x_0 + h$
y_0	$f(x_0, y_0)$	$f(x_1, y_0)$
$y_1 = y_0 + k$	$f(x_0, y_1)$	$f(x_1, y_1)$

A

(a)

Trapezoidal Rule ...Contd

From figure (a)

$$A = \frac{hk}{4}\left[Sum\ of\ \boxed{A}\ square\ terms\right]$$

For *n* number of strips prepare a table as follows

y \ x	x_0	$x_1 = x_0 + h$	$x_2 = x_0 + 2h$	$x_3 = x_0 + 3h$	$x_4 = x_0 + 4h$
y_0	$f(x_0, y_0)$	$f(x_1, y_0)$	$f(x_2, y_0)$	$f(x_3, y_0)$	$f(x_4, y_0)$
$y_1 = y_0 + k$	$f(x_0, y_1)$	$f(x_1, y_1)$	$f(x_2, y_1)$	$f(x_3, y_1)$	$f(x_4, y_1)$
$y_2 = y_0 + 2k$	$f(x_0, y_2)$	$f(x_1, y_2)$	$f(x_2, y_2)$	$f(x_3, y_2)$	$f(x_4, y_2)$
$y_3 = y_0 + 3k$	$f(x_0, y_3)$	$f(x_1, y_3)$	$f(x_2, y_3)$	$f(x_3, y_3)$	$f(x_4, y_3)$
$y_4 = y_0 + 4k$	$f(x_0, y_4)$	$f(x_1, y_4)$	$f(x_2, y_4)$	$f(x_3, y_4)$	$f(x_4, y_4)$

Trapezoidal Rule ...Contd

The total area is given by

$$A = \frac{h}{2}\frac{k}{2}\left[\sum f(x_i, y_i) + 2\sum f(x_i, y_i) + 4\sum f(x_i, y_i)\right]$$

Corner terms Terms at edges Interior terms

$$= \frac{h}{2}\frac{k}{2}\{[f(x_0, y_0) + f(x_4, y_0) + f(x_0, y_4) + f(x_4, y_4)]$$
$$+ 2[f(x_1, y_0) + f(x_2, y_0) + f(x_3, y_0) + f(x_4, y_1) + f(x_4, y_2) + f(x_4, y_3) + f(x_1, y_4)$$
$$+ f(x_2, y_4) + f(x_3, y_4) + f(x_0, y_1) + f(x_0, y_2) + f(x_0, y_3)]$$
$$+ 4[f(x_1, y_1) + f(x_2, y_1) + f(x_3, y_1) + f(x_1, y_2) + f(x_2, y_2) + f(x_3, y_2) + f(x_1, y_3)$$
$$+ f(x_2, y_3) + f(x_3, y_3)]\}$$

The above equation gives the total area under the curve for double integration by Trapezoidal rule.

Trapezoidal Rule ...Contd

Alternate Method:- Prepare a table for n number of strips as follows:

y \ x	x_0	$x_1 = x_0 + h$	$x_2 = x_0 + 2h$	$x_3 = x_0 + 3h$
y_0	$f(x_0, y_0)$	$f(x_1, y_0)$	$f(x_2, y_0)$	$f(x_3, y_0)$
$y_1 = y_0 + k$	$f(x_0, y_1)$	$f(x_1, y_1)$	$f(x_2, y_1)$	$f(x_3, y_1)$
$y_2 = y_0 + 2k$	$f(x_0, y_2)$	$f(x_1, y_2)$	$f(x_2, y_2)$	$f(x_3, y_2)$
$y_3 = y_0 + 3k$	$f(x_0, y_3)$	$f(x_1, y_3)$	$f(x_2, y_3)$	$f(x_3, y_3)$

A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9

Trapezoidal Rule ...Contd

Alternate Method:-

Total area is given by

$$A = \frac{h}{2}\frac{k}{2}[sum\ of\ squares\ A_1 terms + sum\ of\ squares\ A_2 terms + sum\ of\ squares\ A_3 terms + sum\ of\ squares\ A_4 terms + sum\ of\ squares\ A_5 terms + sum\ of\ squares\ A_6 terms + sum\ of\ squares\ A_7 terms + sum\ of\ squares\ A_8 terms + sum\ of\ squares\ A_9 terms]$$

Trapezoidal Rule ...Contd

Example: Evaluate $\int_0^1 \int_0^1 (e^{x+y}) dxdy$ using Trapezoidal rule. Take *h=k=0.5*

Solution: $f(x, y) = (e^{x+y})$, x_0=0, x_n=1, y_0=0, y_m=1, h=k=0.5

Step 1: Calculate the corresponding values of x *and y wrt* h & k

$$x_0 = 0$$

$$x_1 = x_0 + h = 0 + 0.5 = 0.5$$

$$x_2 = x_0 + 2h = 0 + 2(0.5) = 1.0$$

$$y_0 = 0$$

$$y_1 = y_0 + k = 0 + 0.5 = 0.5$$

$$y_2 = y_0 + 2k = 0 + 2(0.5) = 1.0$$

Trapezoidal Rule ...Contd

Step 2: Calculate the values of $f(x_i, y_i)$, $\mathbf{f}(\mathbf{x}, y) = (e^{x+y})$

1) x_0=constant

$$f(x_0, y_0)=(e^{x_0+y_0}) = (e^{0+0}) = 1$$
$$f(x_0, y_1)=(e^{x_0+y_1}) = (e^{0+0.5}) = 1.6487$$
$$f(x_0, y_2)=(e^{x_0+y_2}) = (e^{0+1}) = 2.7182$$

2) x_1=constant

$$f(x_1, y_0)=(e^{x_1+y_0}) = (e^{0.5+0}) = 1.6487$$
$$f(x_1, y_1)=(e^{x_1+y_1}) = (e^{0.5+0.5}) = 2.7182$$
$$f(x_1, y_2)=(e^{x_1+y_2}) = (e^{0.5+1}) = 4.4816$$

Trapezoidal Rule ...Contd

2) x_2=constant

$$f(x_2, y_0)=(e^{x_2+y_0}) = (e^{1+0}) = 2.7182$$
$$f(x_2, y_1)=(e^{x_2+y_1}) = (e^{1+0.5}) = 4.4816$$
$$f(x_2, y_2)=(e^{x_2+y_2}) = (e^{1+1}) = 7.3890$$

<u>Step 3: Make a table for all values of f(x,y)</u>

y \ X	x_0	$x_1 = x_0 + h$	$x_2 = x_0 + 2h$
y_0	$f(x_0, y_0)$	$f(x_1, y_0)$	$f(x_2, y_0)$
$y_1 = y_0 + k$	$f(x_0, y_1)$	$f(x_1, y_1)$	$f(x_2, y_1)$
$y_2 = y_0 + 2k$	$f(x_0, y_2)$	$f(x_1, y_2)$	$f(x_2, y_2)$

.

Trapezoidal Rule ...Contd

x / y	0	0.5	1
0	1	1.6487	2.7182
0.5	1.6487	2.7182	4.4816
1	2.7182	4.4816	7.3890

Step 4: Calculate the area under the curve by Trapezoidal rule

$$A = \frac{h}{2}\frac{k}{2}\{[f(x_0,y_0)+f(x_2,y_0)+f(x_0,y_2)+f(x_2,y_2)]+2[f(x_1,y_0)+f(x_2,y_1)+f(x_1,y_2)+f(x_0,y_1)] + 4[f(x_1,y_1)]\}$$

$$A = \frac{0.5}{2}\frac{0.5}{2}\{[1+2.7182+2.7182+7.3890]+2[1.6487+4.4816+4.4816+1.6487]+4[2.7182]\}$$

Trapezoidal Rule ...Contd

$$A = 0.0625\{13.8254 + 24.5212 + 10.8721\}$$
$$A = 3.0762$$

Step 5: Calculate the area under the curve by alternate method

y \ X	0	0.5	1
0	1	1.6487	2.7182
0.5	1.6487	2.7182	4.4816
1	2.7182	4.4816	7.3890

A_1 A_2 A_3 A_4

Trapezoidal Rule ...Contd

$$A = \frac{h}{2}\frac{k}{2}[sum\ of\ squares\ A_1 terms + sum\ of\ squares\ A_2 terms + sum\ of\ squares\ A_3 terms + sum\ of\ squares\ A_4 terms]$$

$$= \frac{h}{2}\frac{k}{2}\{[f(x_0,y_0) + f(x_1,y_0) + f(x_1,y_1) + f(x_0,y_1)] + [f(x_1,y_0) + f(x_2,y_0) + f(x_2,y_1) + f(x_1,y_1)] + [f(x_0,y_1) + f(x_1,y_1) + f(x_1,y_2) + f(x_0,y_2)] + [f(x_1,y_1) + f(x_2,y_1) + f(x_2,y_2) + f(x_1,y_2)]\}$$

$$= \frac{0.5}{2}\frac{0.5}{2}\{[1 + 1.6487 + 2.7182 + 1.6487] + [1.6487 + 2.7182 + 4.4816 + 2.7182] + [1.6487 + 2.7182 + 2.7182 + 4.4816] + [2.7182 + 4.4816 + 7.3890 + 4.4816]\}$$

$$A = 3.0762$$

Problems on Trapezoidal Rule

1. Find the double integration of $\mathrm{f(x, y)} = x^2 + y^2 + 5$for x=0 to 2 and y=0 to 2 taking increment in both x and y as 0.5 by using Trapezoidal rule.
2. Solve using Trapezoidal rule $\int_0^1 \int_0^1 (x^2y^2)dxdy$ taking step length in x and y as 0.25.
3. Use Trapezoidal rule to evaluate $\int_0^1 \int_1^2 \left(\frac{2xy}{(1+x^2)(1+y^2)}\right) dxdy$.
4. Find the double integration of $\mathrm{f(x, y)}$ =x+y for x=1 to 3 and y=0 to 2 with step size for both x and y as 2 by using Trapezoidal rule.

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What is Integration ?

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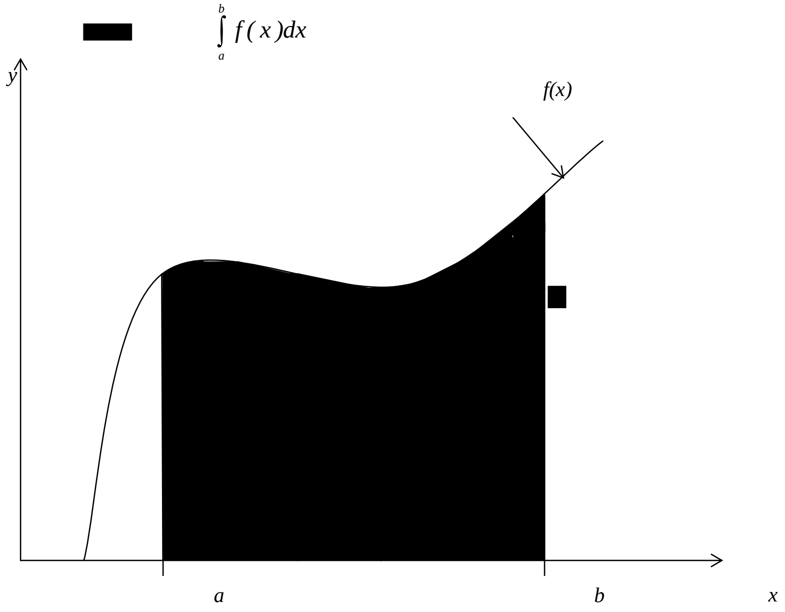
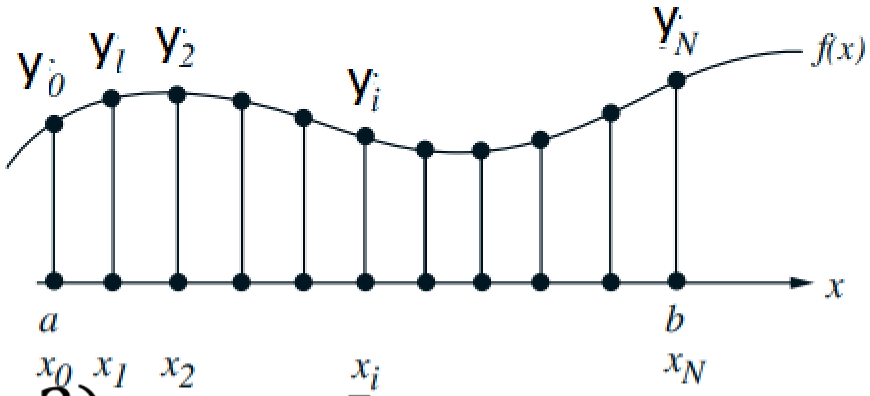
$$I = \int_a^b f(x)dx$$

- Where:
- *f(x)* is the integrand
- a= lower limit of integration
- b= upper limit of integration



Newton-Cotes Quadrature Formula



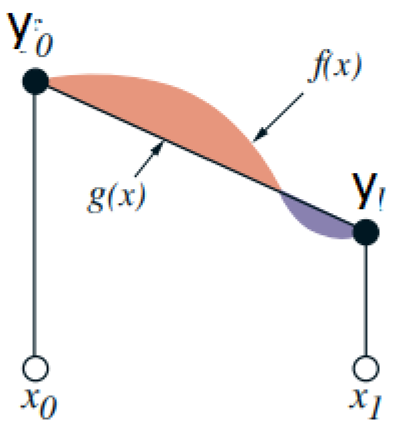
$$I = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \left(\frac{n^4}{5} - \frac{3n^3}{2} + \frac{11n^2}{3} - 3n\right)\frac{\Delta^4 y_0}{24} \dots\dots\dots\dots\dots\dots\dots\right]$$

- Where
- $\Delta y_0 = y_1 - y_0, \Delta^2 y_0 = y_2 - 2y_1 + y_0, \Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$

Trapezoidal Rule

The curve which bounds each strip is approximated as a straight line(first degree polynomial). The Newton-Cotes formula for n=1 becomes

$$I = nh\left[y_0 + \frac{n}{2}\Delta y_0\right]$$



Area under the curve is

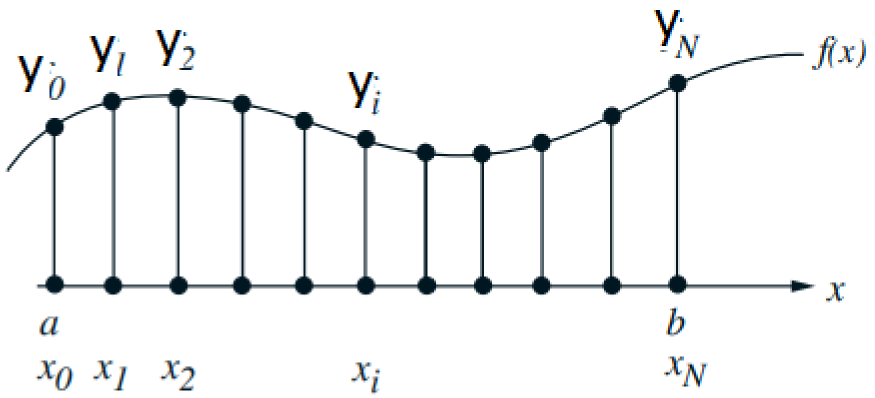
$$A = 1h\left[y_0 + \frac{1}{2}(y_1 - y_0)\right]$$

$$A = \frac{h}{2}[y_0 + y_1]$$

Trapezoidal Rule ...Contd

Consider a curve as shown in figure which is divided in n number of strips and joined by a straight line.

Let A_1, A_2,A_3,....A_n be the area under each strip which is considered as a trapezoid



Area under 1st strip, $A_1 = \frac{h}{2}[y_0 + y_1]$

Area under 2nd strip, $A_2 = \frac{h}{2}[y_1 + y_2]$

Trapezoidal Rule ...Contd

Area under n[th] strip, $A_n = \frac{h}{2}[y_{n-1} + y_n]$

- Total area under the curve is

A = $A_1 + A_2 + A_3 + \ldots. + A_n$

A= $\frac{h}{2}[y_0 + y_1] + \frac{h}{2}[y_1 + y_2] + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{h}{2}[y_{n-1} + y_n]$

A= $\frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots\ldots\ldots\ldots\ldots\ldots + y_{n-1})]$

The above equation is used to calculate area under the curve by trapezoidal rule.

Trapezoidal Rule ...Contd

<u>Example:</u> Evaluate $\int_0^3 (2x - x^2)dx$ taking 6 intervals by Trapezoidal rule.

<u>Solution:</u> $\mathrm{f(x)} = (2x - x^2)$, initial limit x_0=0, final limit x_n=3, n=6

$$h = \left(\frac{x_n - x_0}{n}\right) = \left(\frac{3-0}{6}\right) = 0.5$$

<u>Step 1: Calculate the corresponding values of x *wrt* h</u>

$x_0 = 0$

$x_1 = x_0 + \mathrm{h} = 0 + 0.5 = 0.5$

$x_2 = x_1 + \mathrm{h} = 0.5 + 0.5 = 1.0$

$x_3 = x_2 + \mathrm{h} = 1.0 + 0.5 = 1.5$

Trapezoidal Rule ...Contd

$x_4 = x_3 + \mathrm{h} = 1.5 + 0.5 = 2.0$

$x_5 = x_4 + \mathrm{h} = 2.0 + 0.5 = 2.5$

$x_6 = x_5 + \mathrm{h} = 2.5 + 0.5 = 3.0$

Step 2: Calculate the corresponding values of y *wrt* x

$y_0 = 2x_0 - {x_0}^2 = 2 \times 0 - 0^2 = 0$

$y_1 = 2x_1 - {x_1}^2 = 2 \times 0.5 - 0.5^2 = 0.75$

$y_2 = 2x_2 - {x_2}^2 = 2 \times 1 - 1^2 = 1$

$y_3 = 2x_3 - {x_3}^2 = 2 \times 1.5 - 1.5^2 = 0.75$

$y_4 = 2x_4 - {x_4}^2 = 2 \times 2 - 2^2 = 0$

Trapezoidal Rule ...Contd

$$y_5 = 2x_5 - {x_5}^2 = 2 \times 2.5 - 2.5^2 = -1.25$$
$$y_6 = 2x_6 - {x_6}^2 = 2 \times 3 - 3^2 = -3$$

Step 3: Calculate the area under the curve by Trapezoidal rule

A= $\frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$

A= $\frac{0.5}{2}[(0 - 3) + 2(0.75 + 1 + 0.75 + 0 - 1.25)]$

A= -0.125

$$\int_0^3 (2x - x^2)dx = -0.125$$

Problems on Trapezoidal Rule

1. Find $\int_0^6 (\frac{1}{1+x^2})dx$ using Trapezoidal rule take four strip.
2. UseTrapezoidal rule with four strip to estmate the value of integral $\int_0^2 (\frac{x}{\sqrt{2+x^2}})dx$.
3. Evaluate $\int_4^5 lnx\ dx$ using Trapezoidal rule, take h=02.
4. Find the area under the curve on x axis. The curve passes through the following points (1,2), (1.5,2.4), (2, 2.7), (2.5, 2.8), (3,3), (3.5,2.6), (4,2.10).

Problems on Trapezoidal Rule .....Contd

5. A function f(x) is described by following data. Find numerical integration of the function in limit 1 to 2.2 using trapezoidal rule.

x	1	1.1	1.2	1.4	1.6	1.9	2.2
F(x)	3.123	4.247	5.635	9.299	14.303	24.759	39.319

6. Find the integral $\mathrm{I} = \int_0^{\pi} (sinx)dx$ using Trapezoidal rule. Let h= $\pi/6$.
7. A curve is drawn to pass through the points given by the following table. Find the area under the curve using Trapezoidal rule.

x	1	1.2	1.4	1.6	1.8	2
y	2	2.2	2.7	2.8	3	2.6

8. Evaluate $\int_1^4 (3x^2 + x - 1)\, dx$ by using Trapezoidal rule with 5 strips.

Problems on Trapezoidal Rule .....Contd

9. The total mass of the variable density rod is given by $m = \int_o^L P(x)A_c(x)\,dx$. Where m is mass, P(x) is density, $A_c(x)$ = cross sectional area, x = distance along the rod and L=total length of the rod. The following data is measured for a 10 m length rod. Determine the mass in kg using Trapezoidal rule to best possible accuracy.

x, m	0	2	3	4	6	8	10
P g/cm^2	4.00	3.95	3.89	3.80	3.60	3.41	3.30
A_c, cm^2	100	103	106	110	120	133	150

10. Find the interation of $(4x + 2)$in the limits 1 to 4 by Trapezoidal rule using 6 strips.
11. Find the integration of $\frac{1}{1+x^2}$ in the limit 0 to 1 by Trapezoidal rule using 4 strips.
12. Evaluate $\int_1^4 \left(\sqrt{sinx} + \cos x\right) dx$ by using Trapezoidal rule taking 8 divisions.

Problems on Trapezoidal Rule .....Contd

13. Evaluate $\int_1^4 (e^x + x^3 - 2x + 1)\, dx$ by using Trapezoidal rule taking 6 divisions.
14. Find the interation of $\int_0^2 e^x sinxdx$ by Trapezoidal rule using 4 strips.

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4. Rao V. Dukkipati, Applied Numerical Methods using Matlab, New Age International Publishers

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5. Gerald and Wheatley, Applied Numerical Analysis, Pearson Education Asia

6. E. Balagurusamy, Numerical Methods, Tata McGraw Hill

7. P. Thangaraj, Computer Oriented Numerical Methods, PHI

8. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Thank You